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AN IMPROVED ALGORITHM FOR COMPUTING ALTITUDE DEPENDENT CORRECTED GEOMAGNETIC COORDINATES

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Table of Contents

1. INTRODUCTION 1

2. METHODOLOGY 3

3. GENERATION OF SPHERICAL HARMONIC COEFFICIENTS 5
3.1 GEOCENTRIC TO CGM TRANSFORMATION 5
3.2 CGM TO GEOCENTRIC TRANSFORMATION 5

4. DESCRIPTION OF THE RESULTS 6

5. RECOMPUTING CGM COORDINATES FOR IGRF 1995 AND BEYOND . 12

6. CONCLUSIONS 12

7. REFERENCES 13

8. GLOSSARY AND NOTES 14

List of Figures

Figure — Page

1. Corrected Geomagnetic Coordinate Grid from CGLALO Tables 7
2. Same grid as Figure 1 for Baker routine 7
3. Grid for new routine, at 0 km altitude 8
4. New routine, for 300 km altitude 8
5. New routine at 1200 km altitude 9
6. Consistency of direct and inverse conversion 9
7. Consistency: Direct and inverse at 300 km altitude 10
8. Consistency: Direct and inverse at 1200 km altitude 10

List of Tables

Table — Page

1. Inversion Error Analysis: Geocentric to CGM to Geocentric 11
2. Inversion Error Analysis: CGM to Geocentric to CGM 11

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The CGLALO90 line traced tables were verified using program GEOCGM which is due to the Papitashvili's and Gustafsson.

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1. INTRODUCTION

The motion of charged particles in the ionosphere and in the radiation belts is determined by Earth's magnetic field. For this reason, it is desirable to use the geomagnetic domain for correlation with observations of charged particles. For near Earth applications, these particle locations are therefore expressed in Earth fixed geocentric coordinates, using a simple 6371.2km radius "spherical Earth" model, which differs slightly in altitude and latitude from the usual geodetic "oblate Earth" model. In the early period of satellite exploration of the magnetosphere, the use of a coordinate system, geomagnetic coordinates, based upon a centered but tilted dipole representation of Earth's magnetic field was often sufficient for most applications. As more precise measurements became possible, the need arose for a coordinate system which would more closely represent the actual magnetic field. In 1958 Hultqvist published two papers [*Hultqvist*, 1958a; 1958b] defining a corrected magnetic coordinate system taking into account higher order terms in the spherical harmonic expansion of the 1945 magnetic field model. A real field line from Earth's surface may be traced to the centered dipole equator. This point is next defined to be equivalent to a line trace along a centered dipole field. The latitude and longitude of the point in dipole coordinates are the desired "corrected" geomagnetic coordinates.

In 1965 Hakura used the higher order terms in the spherical harmonic expansion of Earth's magnetic field to compute tables and maps of corrected geomagnetic coordinates. Since the actual magnetic field changes with time, it is necessary to generate new tables and maps. *Gustafsson* [1970 and 1984] has provided revised corrected geomagnetic coordinates based upon the International Geomagnetic Reference Field Epoch 1965 and Epoch 1980 (or IGRF65 and IGRF80). The earlier paper defines a set of hypergeometric functions which may be used to compute the corrections for a spherical harmonic representation of the magnetic field of order up to 7. Line tracing with modern computers makes the analytic approach unnecessary while permitting the use of higher order spherical harmonic field representations. More recently *Gustafsson, et al.* [1992] have performed similar calculations for the IGRF 1990 magnetic field model. The corrected geomagnetic coordinates provided in the tables described above are for 0 km altitude. Using this definition of CGM, there are areas of Earth's surface where magnetic field line traces never reach the dipole equator plane. *Gustafsson, et al.* [1984] describes various interpolation methods to fill in these forbidden areas.

True corrected geomagnetic coordinates are defined only at ground level, but a method is needed to provide geomagnetic coordinates at all altitudes. This report describes the development and implementation of such a method, which properly should be called "altitude adjusted corrected geomagnetic coordinates". Introduction of new terminology appears unwarranted however, and the altitude dependent conversions as well are referred to as corrected geomagnetic coordinates. Since the same procedure of field line tracing to the Earth-centered dipole equator applies, all points along a field line (on one side of the magnetic equator) have the same corrected geomagnetic coordinates (CGM). In practice, for non-zero altitudes, the actual approach taken is to trace down to zero altitude, and to then look up conventional CGM coordinates and interpolate using the tables printed in the above references. For IGRF90 the look up and interpolation procedure has been automated

in a routine which we shall refer to as CGLALO90.

For non-zero altitudes at or near the magnetic equator, the field lines trace down to higher geomagnetic latitudes. The higher the altitude, the greater is the separation of the foot of the field line from the dip equator, so that low CGM latitudes do not exist for non-zero altitudes, and a significant discontinuity in latitude is present.

In a recent paper, *Baker and Wing* [1989] describe an alternative method to compute corrected geomagnetic coordinates. In their method, the corrected geomagnetic coordinates (and the corresponding inverses) are computed by evaluation of functions for the X, Y, and Z components of a unit vector obtained from a fourth order spherical harmonic expansion. They first computed the coefficients for the X, Y, Z components of a unit vector in the magnetic dipole coordinates for both the forward and inverse transformations for altitudes 0, 150, 300 and 450 km. They developed an interpolation/extrapolation scheme for computing the spherical harmonic coefficients for altitudes from 0 to 2000 km altitude. Since they were primarily concerned with representing higher CGM latitudes, the equatorial problem described above was not taken into account. As a result, the features of the South Atlantic Anomaly and the equatorial region are not well represented in the Baker and Wing computation. The spherical harmonic expansion computations were performed in dipole magnetic coordinates. Coordinate conversions between geographic and dipole coordinates were accomplished by using rotation matrices.

In this report, we will describe an improved method to calculate the CGM coordinates and their inverses at altitudes from 0 to 2000 km. This approach is similar in many respects to that of Baker and Wing, but provides an improved representation of the South Atlantic Anomaly region, and a solution to the equatorial discontinuity problem. In Section 2 we will describe the methodology used for the solution of this problem. In Section 3 we will provide a detailed description of the computation of the spherical harmonic coefficients. In Section 4 we will describe the results obtained using this method, together with graphs.

2. METHODOLOGY

For the analysis of satellite observations having a circular or near circular orbit, the use of CGM tables and interpolation methods for a fixed altitude, such as found in CGLALO90, are well suited. For satellites with more eccentric orbits, and other applications at non-uniform altitudes, a functional representation in terms of a spherical harmonic expansion, such as implemented by Baker and Wing is more suitable, because a single routine can inherently interpolate smoothly over the entire region of space of interest. The equatorial discontinuity problem is handled by using an auxiliary coordinate system (magnetic dipole coordinates at altitude) to compute the spherical harmonic coefficients which are incorporated into the code. A simple mapping is used to transform to and from dipole coordinates at altitude and dipole coordinates at 0 km altitude. The lengthier calculations involved are usually not a burden with modern computers, and computational efficiency techniques can be incorporated when working at a constant altitude.

In using a spherical harmonic representation of a function defined on a spherical surface, where the function is initially specified by a table of values, there must be sufficient data in the table, and the order of the spherical harmonic expansion must be chosen to adequately represent the function at the tabulated values.
The spherical harmonic coefficients for a function f, $a_{l,m}$, are usually computed from the following integrals.

$$a_{l,m} = \int_{\Omega} f(\theta,\phi)\ Y_{l,m}(\theta,\phi)\ d\Omega \qquad \textbf{(1)}$$

This is the approach used in this study. To compute these integrals it is necessary to have a completely defined uniform grid. We found that a table of values between -88 and 88 degrees latitude at 2 degree intervals, and 0 - 350 degrees in longitude at ten degree intervals is adequate for a tenth order spherical harmonic expansion. All the computations described here were made with such a coordinate grid.

A significant aspect of spherical harmonic fitting is the problem of convergence. In the theory of Fourier series, there is a problem with the convergence of the partial sums of the Fourier expansion for a function in the vicinity of a discontinuity. A typical example is the case of a step function, in which the partial sums oscillate in the vicinity of the jump. Similar, but less pronounced behavior, is found when the function to be represented is continuous, but has a discontinuous first derivative.

To avoid this problem (Gibbs' phenomenon), the functions chosen for the spherical harmonic

expansion must be a periodic function in longitude (or its equivalent) and have no discontinuities. For that reason it is not practical to use the longitude variable itself. The simplest reasonable choice of functions are the complex exponentials exp iθ and exp iϕ (or their real two dimensional vector equivalents) where θ and ϕ are the co-latitude and longitude respectively. However, using this approach, problems arise with the quality of the spherical harmonic expansion fit to the actual data near the magnetic poles. We choose the unit vector approach used by Baker and Wing because the spherical harmonic expansion fit does not exhibit any pathology in the vicinity of the magnetic poles.

Since CGM values for non-zero altitudes have a discontinuity near the magnetic equator, it is not practical to use a ground-based dipole coordinate system for either computing the spherical harmonic coefficients or the spherical harmonic expansion. We used an at-altitude dipole-coordinate system at the 300 and 1200 km altitudes to perform these computations. The altitude dependent mapping described above to transform between the actual CGM latitude λ_{CGM} and an at-altitude dipole latitude λ_{dipole} is given by:

$$\cos^2 \lambda_{dipole} = \left(1 + \frac{altitude[km]}{6371.2}\right) \cos^2 \lambda_{CGM} \qquad \textbf{(2)}$$

The use of the at-altitude dipole coordinate system given by the above transformation (and its inverse) effectively "closes" the discontinuity, permitting the calculation of the spherical harmonic coefficients at the 300 and 1200 km altitudes.

To compute the spherical harmonic expansion at arbitrary altitudes between 0 and 2000 km, the spherical harmonic coefficients for 0, 300 and 1200 km were fit to a quadratic (using altitude/1200 as the independent variable); the actual coefficients provided in the program contain the constant (altitude = 0 km), linear and quadratic terms obtained from the quadratic fit. The extension from 1200 to 2000 km constitutes a safe extrapolation, but remains to be verified.

For the computation of the geocentric to CGM coordinates, the procedure used was as follows:

(1) Compute (if new altitude was different from the last) the altitude dependent spherical harmonic coefficients.

(2) Compute the spherical harmonic expansion for the X, Y, Z components of the unit vector describing the orientation of the transformed point in the altitude dependent dipole coordinate system.

(3) Compute latitude and longitude of point, and apply the altitude transformation to the at-altitude dipole latitude. Return the computed corrected latitude and longitude values.

For the inverse computation, the procedure used was as follows:

(1) Compute (if new altitude was different from the last) the altitude dependent spherical harmonic coefficients for the inverse calculation.

(2) Transform the CGM input latitude to the at-altitude dipole latitude. Set error return flag and default return value if input latitude was invalid.

(3) Compute the spherical harmonic expansion for the X, Y, Z components of the unit vector describing the orientation of the transformed point in geocentric coordinate system.
Return the computed geocentric latitude and longitude values.

3. GENERATION OF SPHERICAL HARMONIC COEFFICIENTS

The spherical harmonic coefficients for the 0, 300 and 1200 km altitude fits for both transformations were obtained using the tables described below using the standard formulas for computing spherical harmonic coefficients. The spherical harmonic coefficients were computed for the components of a unit vector in the target coordinate systems, as defined by the direction cosines.

3.1 GEOCENTRIC TO CGM TRANSFORMATION

For 0 km altitude, the Gustafsson et al tables (corrected for misprints) were used except for a band of ~15 degrees around the magnetic equator. For the equatorial region, a spline fit was made to fill in the missing grid points, with the added requirement that the spline curve geocentric latitudes obey the dip equator constraint at all longitudes. The modified table was incorporated into a new version of a FORTRAN subroutine (CGLALO90) which uses a look-up and interpolation procedure to compute CGM coordinates for arbitrary points at 0 km altitude. For 300 and 1200 km, the field lines for the IGRF90 magnetic field model were traced from altitude to ground using a precise field line trace routine. The new version of CGLALO90 was then used to compute the CGM tables for the respective altitudes.

3.2 CGM TO GEOCENTRIC TRANSFORMATION

For the inverse transformation, a new version of the CGLALO inverse routine was written. This routine uses a search procedure to locate a point within a spherical "rectangle" (or, if the magnetic pole lies inside the rectangle, within a spherical "triangle" with the apex at the magnetic pole), and uses calls to the new version of CGLALO90 to perform the required interpolations.

For 0 km altitude, the desired table was computed for a standard grid of CGM coordinates using the

new CGLALO inverse routine. For 300 and 1200 km, the input grid CGM latitudes were converted to dipole coordinates (at altitude) as described earlier. The tables were generated for the coordinate grid (dipole coordinates at altitude) using the CGLALO inverse routine.

The six tables (three for geocentric to CGM, and three for the inverse) were used to generate the required spherical harmonic expansion coefficients for 0, 300 and 1200 km altitudes.

4. DESCRIPTION OF THE RESULTS

The resulting new code provides a substantial improvement in the representation of the equatorial region, while retaining excellent agreement with the Gustafsson tables at the poles and at medium latitudes. Figure 1 is a graph of the CGM coordinates from the corrected Gustafsson et al [1992] IGRF 1990 model tables. Figure 2 provides a similar graph from the Baker/Wing code which was also computed from similar Gustaffson's tables for epoch 1987. Figures 3, 4 and 5 are graphs of the CGM coordinate at 0, 300 and 1200 km altitude obtained from the new code. In all these graphs, the data was generated using a two degree geocentric latitude interval between -88° and 88° and a ten degree interval in longitude at 0, 300, and 1200 km respectively. In Figures 4 and 5 there is a marked bending of the constant longitude curves at the edge of the equatorial gap.

Ideally, the output of the Geocentric to CGM calculations, fed into the inverse computation, should be identical to the original input coordinate grid. A test of the consistency of the direct and inverse transformations is illustrated in Figures 6, 7 and 8 for 0, 300 and 1200 km altitude. These graphs exhibit deviations from the uniform spacing (2 degrees in latitude, 10 degrees in longitude) of the original grid, particularly in the vicinity of the poles, and in the vicinity of the South Atlantic Anomaly. The longitude difference at the poles loses significance, and a more accurate measure is the great circle arc between the coordinate pairs. Tables 1 and 2 provide the fraction of table values which lie in the following error intervals (in degrees) for the Geocentric ==> CGM ==> Geocentric and (where they exist) the CGM ==> Geocentric ==> CGM coordinates respectively:

$$0° - 0.1°,\ 0.1° - 0.2°,\ 0.2° - 0.5°,\ 0.5° - 1.0°,\ 1.0° - 2.0° \text{ and } > 2.0°.$$

Thus, except for problems near the South Atlantic Anomaly, and near the "forbidden" band at altitude, the new algorithm performs adequately.

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Table 1.
Inversion Error Analysis
GEOCENTRIC ==> CGM ==> GEOCENTRIC
FRACTION OF ERRORS IN RANGE [deg]

ALT [KM]	0.0-0.1	0.1-0.2	0.2-0.5	0.5-1.0	1.0-2.0	> 2.0
.0	.54557	.30400	.10549	.02871	.01030	.00593
100.0	.49001	.30868	.14139	.03527	.01248	.01217
200.0	.44944	.28620	.19039	.03683	.01904	.01810
300.0	.42104	.27091	.21816	.04494	.02559	.01935
400.0	.39950	.26561	.22846	.05368	.03027	.02247
500.0	.38046	.26248	.23939	.06211	.03090	.02466
600.0	.36642	.26311	.24345	.06898	.03340	.02466
700.0	.36174	.26685	.24095	.07085	.03464	.02497
800.0	.36954	.26436	.23439	.07303	.03402	.02466
900.0	.38171	.26217	.22940	.07179	.03152	.02341
1000.0	.40605	.25843	.21879	.06898	.02622	.02154
1100.0	.43477	.25687	.20724	.05743	.02622	.01748
1200.0	.47066	.24657	.19913	.05025	.01873	.01467
1300.0	.50718	.24938	.17416	.04650	.01373	.00905
1400.0	.55243	.25499	.13608	.03995	.01280	.00375
1500.0	.53433	.28652	.12360	.04151	.01404	.00000
1600.0	.46910	.32584	.13015	.05181	.02247	.00062
1700.0	.37266	.34644	.17665	.06305	.03402	.00718
1800.0	.27965	.32772	.25499	.06960	.04744	.02060
1900.0	.19257	.31461	.30774	.09738	.05431	.03340
2000.0	.12797	.28277	.34519	.13577	.05524	.05306

Table 2.
Inversion Error Analysis
CGM ==> GEOCENTRIC ==> CGM
FRACTION OF ERRORS IN RANGE [deg]

Note: Points for which the CGM to Geoc. input values were not valid were excluded.

ALT [KM]	0.0-0.1	0.1-0.2	0.2-0.5	0.5-1.0	1.0-2.0	> 2.0
.0	.58365	.26799	.11111	.02399	.01294	.00032
100.0	.53286	.28320	.14228	.02846	.01253	.00068
200.0	.49608	.28170	.17415	.03170	.01389	.00249
300.0	.46857	.27632	.19956	.03399	.01864	.00292
400.0	.44670	.27665	.21396	.04129	.01802	.00338
500.0	.43994	.27177	.21734	.04505	.01839	.00751
600.0	.42785	.27662	.22222	.04900	.01929	.00502
700.0	.42421	.27540	.22659	.04960	.02024	.00397
800.0	.43929	.27143	.22024	.04643	.01984	.00278
900.0	.46038	.25980	.21324	.04739	.01757	.00163
1000.0	.49265	.24632	.20588	.04126	.01348	.00041
1100.0	.51599	.24663	.18561	.04461	.00715	.00000
1200.0	.55303	.24453	.16120	.03746	.00379	.00000
1300.0	.59462	.23655	.14106	.02474	.00304	.00000
1400.0	.65017	.22700	.10590	.01519	.00174	.00000
1500.0	.63759	.25911	.08247	.01823	.00260	.00000
1600.0	.55152	.32348	.08961	.03091	.00448	.00000
1700.0	.44758	.35977	.14113	.04167	.00986	.00000
1800.0	.34857	.33468	.23835	.05735	.01613	.00493
1900.0	.25278	.32269	.32315	.06759	.02870	.00509
2000.0	.18935	.30509	.35324	.10324	.04167	.00741

5. RECOMPUTING CGM COORDINATES FOR IGRF 1995 AND BEYOND

Assuming the use of the same algorithm, and the same order spherical harmonic expansion, an update to the new CGM computation would only require the replacement of the existing six sets of spherical harmonic expansion coefficients with new sets corresponding to the new magnetic field model for the direct and inverse transformations at 0, 300 and 1200 km. The procedure for computing the new sets of coefficients was described in Section 2. In the FORTRAN code implementation of the new algorithm, this would be accomplished by replacing the existing BLOCKDATA source code containing the expansion coefficients table with a new table, and recompiling the source code with the replacement BLOCKDATA tables.

6. CONCLUSIONS

The Cartesian spherical harmonic approach of *Baker and Wing* [1989] for geocentric to corrected geomagnetic conversion (and the inverse) has been enhanced by introducing auxiliary coordinates (dipole coordinates at altitude) that are derived by applying a basic altitude adjustment algorithm to the CGM latitudes. In this auxiliary coordinate system the magnetic equator discontinuity described above is eliminated, permitting accurate fitting to 10th order spherical harmonic expansions. These coordinates also overlay closely for altitudes up to 2000 km, allowing a simple quadratic fit to each coefficient with altitude.

There are however regions, particularly in the vicinity of the South Atlantic Anomaly, for which better representation is desirable. An offset auxiliary altitude adjusted coordinate system should further ameliorate the irregularities, but this remains to be implemented.

The present effort employed the IGRF90 model for the required line tracing, as well as to define the dipole equator. The coefficients that are employed do not include secular terms, and an updated model is warranted every few years.

Corrected geomagnetic coordinates are linked to field line tracing, and are thus used extensively for particle mapping. There is a need to extend this application to outline ionospheric effects at low latitudes and, in this respect, the current CGM system is deficient because points around the geomagnetic equator on Earth's surface at various longitudes differ significantly in L-shell or "geomagnetic altitude". Recognizing this, a corrected eccentric geomagnetic coordinate system, better designed for global ionospheric mapping, could be formulated.

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8. GLOSSARY AND NOTES

Earth's magnetic field arises from contributions both within and external to it. For most near Earth applications (typically 1 Earth radius), the external field may be ignored. The internal field is described in terms of its geomagnetic potential, and is available in mathematical form as spherical harmonic coefficients and their secular variations. This model is the responsibility of the International Association of Geomagnetism and Aeronomy (IAGA), and is published periodically as revisions to the International Geomagnetic Reference Field (IGRF). Those more interested in the history and development of this subject are referred to the classic book *Geomagnetism* [*Chapman and Bartels*, 1940] which in turn describes the original work of Gauss, Schmidt who introduced Geomagnetic Coordinates, and many others.

Ionospheric phenomena near Earth are intimately controlled by the Earth fixed geomagnetic field and, because of the substantial and unnecessarily repetitive calculations involved in field line tracing, simplified models and procedures become essential. Although the dipole and the offset dipole models can be determined directly from the first and second order terms of the IGRF spherical harmonics and are useful for conceptual purposes, field line traces cannot be inferred with adequate accuracy from these models. Fortunately, the internal geomagnetic field is Earth fixed, and extensive a priori computations can be carried out to provide tables which relate geographic locations to their corresponding field line trace environment. This approach was used by *Hultqvist* [1958a, 1958b] to define and introduce Corrected Geomagnetic Coordinates, and subsequently revisions were made by *Hakura* [1965] and *Gustafsson* [1970]. Their work defines these coordinates with tables at the surface of Earth only. Later work leading to our present effort is described in the main text of this report.

Below we describe many of the terms covered or related to the present work. The asymmetrical nature of the geomagnetic field has given rise to the need for dipole, eccentric (or offset) dipole, corrected geomagnetic, and dip-pole representations, all of which are distinct in some manner. Thus, for instance, geomagnetic field lines are not truly perpendicular to Earth's surface at the corrected geomagnetic poles, but rather at the dip-poles. The reader should also be aware that the field undergoes a secular variation, and the assorted magnetic poles migrate one to a few kilometers per year.

Altitude Dependent Corrected Geomagnetic coordinates:
Extension of Corrected Geomagnetic Coordinates to altitudes above Earth's surface. Defined so that all points along a field line possess the same coordinates. Not part of Hultqvist's original definition of CGM coordinates.

Corrected Geomagnetic (CGM) coordinates:
Earth fixed magnetic latitude and longitude. Altitude is undefined. Prescribed by Hultqvist and Gustafsson and used in this report. Entails tracing along field lines to the dipole equator, and then determining the geomagnetic coordinates corresponding to this point on the dipole equator as if it had been reached by tracing along a pure dipole. Zero corrected geomagnetic longitude is the meridian which passes through the geographic South Pole, with East positive.

Corrected Geomagnetic (CGM) coordinate Poles:
Locations in the polar regions from where internal geomagnetic field line traces effectively intercept the dipole equatorial plane at an infinite distance. For Epoch 1990.0 on which the present report is based, the north and south corrected geomagnetic poles are at 81.0°N latitude and 278.5°E longitude, and at 74.0°S latitude and 126.0°E longitude, respectively. Inversely incidentally, the corrected geomagnetic coordinates of the geographic north and south poles are at 82.30°N latitude and 170.89°E longitude, and at 73.89°S latitude and 18.55°E longitude, respectively.

Dip Equator:
The plane at low latitudes where Earth's field becomes horizontal, so that the magnetic dip angle is zero. This resolves the problem with the Hultqvist procedure of tracing to the dipole equator, which results in imaginary latitudes when the field line terminates inside 1 Earth radius. Field lines undulate and the geographic latitude corresponding to zero corrected geomagnetic latitude sometimes had to be estimated by curve fitting.

Dip Poles:
North and south polar locations where the geomagnetic field at Earth's surface is vertical. Roughly at 78°N latitude and 256°E longitude, and at 65°S latitude and 139°E longitude, respectively.

Dipole:
Simple first order Earth centered representation of geomagnetic field. For epoch 1990, the first order X,Y,Z dipole moments are 1851, -5411, 29775 nanoTesla respectively, which places the dipole north magnetic pole roughly at 79°N latitude and 289°E longitude, and the south magnetic pole at 79°S latitude and 109°E longitude. The plane through the center of Earth normal to this axis determines the dipole equator.

Dipole Equator or Eccentric Dipole Equator:
Since the eccentric offset is roughly in the plane of the pure Dipole equator, the same equatorial plane through Earth's center, normal to the axis of the poles, applies to Dipole or to Eccentric Dipole. See *Dipole* and *Eccentric Dipole*.

Dipole Poles:
North and south intercepts of dipole axis with Earth's surface. See *Dipole*.

Eccentric (or offset) Dipole:
Field lines do not trace out normal to Earth's surface at the dipole poles, but considerably removed particularly for the South magnetic pole, which implies an eccentric rather than a centered dipole. The next few terms in the representation of Earth's field in IGRF90 suggest an offset dipole centered approximately at geocentric X,Y,Z rectangular coordinates -400, 270, 190 km respectively, roughly in the direction of the Marianas Trench and farthest removed from the South Atlantic Anomaly.

Eccentric Dipole Poles:
North and south intercepts of eccentric dipole axis with Earth's surface, roughly at 82°N latitude and 259°E longitude, and at 75°S latitude and 119°E longitude, respectively. See *Eccentric Dipole*.

Geocentric:
Earth centered. Since latitude is defined by the angle between the vector to the location and the equatorial plane, geocentric coordinates imply a spherical Earth model. Magnetic field models, such as the International Geomagnetic Reference Field (IGRF) use 6371.2 km as the mean Earth radius to normalize radial distance and, for convenience, geocentric altitude refers to this radius when describing particle locations and the geomagnetic environment.

Geodetic:
Refers to oblate Earth and the local horizontal plane. Not used or implied in this report.

Geographic coordinates:
Earth fixed latitude, longitude, and altitude. Although commonly loosely applied to geodetic or geocentric, the spherical 6371.2 km radius of Earth applies throughout this report, with all latitudes, altitudes, and field line trace terminations determined by this model. Geodetic and geocentric longitudes are identical, with 0° passing through the Greenwich meridian, and East positive.

Geomagnetic coordinates:
Earth fixed magnetic dipole latitude and longitude. Altitude is undefined. The pure dipole axis is tilted with respect to Earth's axis and the poles approach the magnetic poles. Zero geomagnetic longitude is the geomagnetic meridian which passes through the geographic South Pole, with East positive.

Inverse Coordinate Conversion:
Obtains geographic coordinates, given CGM coordinates. The reverse of the geographic to CGM coordinate conversion. Since the conversions are altitude dependent, the altitude at which the geographic coordinates are desired must be specified. Except for the fitting approximations arising from analytical modeling, inversion following a coordinate conversion should return to the original latitude and longitude.